FUZZY INTERPRETATION OF SYSTEMS OPERATION EVALUATION

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Abstract

The main subject of this paper is the analysis of possibility of fuzzy logic elements implementation in the field of systems operation evaluation.

The analysis considers complex systems exploitation process. The process is evaluated on a base of the system features values. The changes of system features values describe the quality of system operators' activity, technical objects and surrounding environment influence. The quality of system operation is the function of exploitation process safety, efficiency and reliability. The evaluation and assurance of required quality of system operation is the main factor in exploitation process.

On the base of created complex method of systems operation evaluation the fuzzy model was built. The sharp values of system features are the input parameters of the model. In this model the range of analyzed system features was partitioned by fuzzy digits. In the paper the fuzzy representation of system features was presented. The multidimensional and model vector of system operation quality is also described. The highest system operation quality presents the case when these spaces have not empty intersection set. The lower quality is when spaces are tangential. The lowest quality is when the spaces have empty intersection set.

The pattern of systems operation quality and their current evaluation in the moment t is calculated on the base of the points in n-dimensional space. The position of the spaces is described by the rages of systems features values. The features are the functions of the time. To model all these situations in the paper the evaluation of system operation quality is interpreted as multi objective analysis problem soled by fuzzy sets theory implementation.

Keywords: *quality of operation, system, assessment method, fuzzy model*

1. The method of the system operation quality

On the basis of the literatury analysis and the curried out studies the quality of system operation was defined as *a set of the system features, expressed by their number values in the given moment of time t, which determines the level of the formulated requirements fulfilment* [9].

To assess the system operation quality the external observer OZ identifies the object of the studies and on the basis of that determines the set of the features X={ $X_1,X_2,\ldots,X_i$}. The set describes the system from the operation quality point of view. The observer, on the basis of received results, determines the domain and the minimum and maximum values of the feature. Next the set of the criteria K_i $i=\overline{1,n}$, is established. The set of the criteria includes the sub-criteria k_{ij} $j=\overline{1,r}$ which includes the factors of the features assessment important in the given moment t, $t\in(t_0,t_k)$. At the end the rates of the features values are determined and the level of the criteria fulfilment is checked.

The criterion is defined as *a one from the important conditions applied to the features value determining the quality of the analysis object in given moment t. The feature is defined as a* *property or characteristic of the analysed object. The property is the common feature (phisical value) of all considered objects. The characteristic is a feature which enables to distinguish the objects between each other* [7].

It should be noticed that the set of the features consists of the measurable and not measurable features. Not measuable features couldn't be measured because of the technical problems or the lack of the expert knowleage.

For each measurable feature X_{Mi}, (i=1,2,...,n) it is necessary to specify the maximum and the minimum values $X^{min}_{M,i}$, $X^{max}_{M,i}$. They deternime the correct (desired) quality of the system operation. Similarly, for each not measurable feature X_{Nj}, (j=1,2,...,m) it is necessary to establish the conditions of the correct system operation. The conditions are defined to enable the assessment the feature value. To do it, for each not measurable feature, the values from 0 to m are assigned. So, the condition of the correct system operation in given moment t, $t \in [t_0, t_k]$ could be expressed in the following form:

$$J_S = \begin{cases} X^{min}_{M,1} < X_{M,1,t} < X^{max}_{M,1} \ldots X^{min}_{M,n} < X_{M,n,t} < X^{max}_{M,n,} \\ X^{min}_{N,1} < X_{N,1,t} < X^{max}_{N,1} \ldots X^{min}_{N,m} < X_{N,m,t} < X^{max}_{N,m} \end{cases}. \tag{1}$$

The formula means that in given moment t the quality of the system operation is on the acceptable level only when the values of the measurable features are between defined extreme values and not measurable features values fulfil the defined conditions of the correct system operation.

2. Fuzzy representation of the system features values

The set of the system features can be divided into two groups. The first group consists of the features described in form of continuous measured values. The second one consists of the features assessed in the digital way. To combain both types of the features in one coherent assessment system the fuzzy modelling could be implemented [4]. In case of the measured features the value is determined with the accuracy of the measure device [8]. So it is not possible to specify the value precisely. It is only possible to determine the interval which covers the value:

$$X_o(t) \in \left\langle X_p(t) - \delta_u, X_p(t) + \delta_u) \right\rangle, \tag{2}$$

where:

$X_o(t)$ - calculated value of the feature,

$X_p(t)$ - measured value of the feature,

δ_u - accuracy of the measure device.

Additionally in case of the indirect measurements the value is burden with the inaccuracy of the calculation method. Because of that it is necessary to consider the interval of the tolerance of the value. During the studies the interval of tolerance is expessed in form of fuzzy set.

In case of the measurements with the insensivity zone the tolerance interval was model by the *Π* type fuzzy set.

$$FS_\Pi(x) = \begin{cases} 0 & \Leftrightarrow x \le lrs \vee x \ge rrs \\ \dfrac{x - lrs}{lrk - lrs} & \Leftrightarrow lrs < x \le lrk \\ 1 & \Leftrightarrow lrk < x \le rrk \\ \dfrac{rrs - x}{rrs - rrk} & \Leftrightarrow rrk < x < rrs \end{cases}, \tag{3}$$

where:

$FS_{\Pi}(x)$ – member function of *Π* type fuzzy set,

lrk – the lowest value of the fuzzy set kernel,

lrs – the lowest value of the fuzzy set support,

rrk – the biggest value of the fuzzy set kernel,

rrs – the biggest value of the fuzzy set support.

In the remaining cases the tolerance interval was modeled in form of the *Λ* type fuzzy set [3]:

$$FS_{\Lambda}(x) = \begin{cases} 0 & \Leftrightarrow x \le lrs \vee x \ge rrs \\ \dfrac{x - lrs}{lrk - lrs} & \Leftrightarrow lrs < x \le lrk \\ \dfrac{rrs - x}{rrs - lrk} & \Leftrightarrow lrk < x < rrs \end{cases}, \tag{4}$$

where:

$FS_{\Lambda}(x)$ - member function of *Λ* type fuzzy set,

lrk – the lowest value of the fuzzy set kernel,

lrs – the lowest value of the fuzzy set support,

rrs – the biggest value of the fuzzy set support.

As a modal value of the fuzzy set the measured value was admitted. The support of the fuzzy set is equal to the sharp interval of the considered tolerance.

In case of the features enumered between the elements of the second of the mentioned above sets the assessment could be subjective. Additionally, the assessment is done with the approximation resulted from the frequency of the discretisation. The inaccuracy was modeled using the *Λ* type fuzzy sets. For the fuzzy set the criterion fulfilment level was established as the modal value. The extent of the fuzzy set support is equal to the distance between the discret values of the criterion assessment scale multiplied by two.

3. Multi-objective assessment method of the system operation quality

During the studies the system operation quality assessment process is considered as the multi-objective analysis issue. For each ystem feature the criterion was formulated. The fulfilment level of the criteria describes the quality of the system operation.

According to the implemented method, the domain of each criterion was defined on base of the criterion argument extent. The domain was divided into six intervals. The size of interval increases according to the geometric series as a distance from the optimum value. The quotient of the series equals to *2* [5]. Different types of the criteria were expressed by different functions. The criterion where the optimal value is the smallest one and the most important differences in level of criterion fulfilment are around the minimum point of the criterion domain was expressed by the function:

$$\nu = \log_2\left(\frac{P_\nu - P_{min}}{P_{max} - P_{min}} \cdot 64\right). \tag{5}$$

The criterion where the optimal value is the biggest one was expressed by the function:

$$\nu = \log_2\left(\frac{P_{max} - P_\nu}{P_{max} - P_{min}} \cdot 64\right). \tag{6}$$

For the criterion described by the function (5) the function values respond to the arguments values determined by the formula (7),

$$P_\nu = P_{\mathbf{min}} + \left(P_{\mathbf{max}} - P_{\mathbf{min}}\right)\cdot\frac{2^\nu}{64}, \nu = 0,1,\ldots,6 \tag{7}$$

in range from *4* to *10* according to (8)

$$g = 10 - \nu\,. \tag{8}$$

For the criterion described by the function (6), the function values respond to the arguments values determined by the formula (9) and the range is described by (10) [1]

$$P_\nu = P_{\mathbf{max}} - \left(P_{\mathbf{max}} - P_{\mathbf{min}}\right)\cdot\frac{2^\nu}{64}, \nu = 0,1,\ldots,6\,, \tag{9}$$

$$g = 4 + \nu\,. \tag{10}$$

where:

ν - criterion function value,
P_{max} - maximum value of the criterion argument,
P_{min} - minimum value of the criterion argument,
P_ν - the argument of the criterion,
g - fulfilment level of the criterion.

As it was mentioned above the method enables to create the coherent assessment system that take into consideration both types of the features. It could be also noticed that the features of the system could be defined in different domains. Described method transform all the criteria to one domain. Thanks to it the total assessment could be calculated.

During the studies it was recognized that because of the fuzzy characteristic of the system features it is not possible to establish the accurate level of the criteria fulfilment. To do it possible the fuzzy extension of the method was developed. The extension describes each criterion by the fuzzy set where the member function determines the criterion fulfilment level. The domain of the criterion is equal to the fuzzy set support. The member functions for the criteria were appointed as a linear interpolation of the points described by the formulas (5-10). The functions were scaled to the fuzzy set domain by the fuzzy set division into *64* equal parts.

The values of the functions are included in the range from *4* to *10*. So, they are transformed to create the normal fuzzy sets:

$$\sup \mu_{FS}(X_i) = 1, \tag{11}$$

where:

$\mu_{FS}(X_i)$ - member function for system feature no. *i*.

To do it the values of the function were divided by 10.

Thanks to described method implementation it is possible to carry out the system operation quality assessment process taking into consideration the inaccuracy of the system feature values.

4. Graphic interpretation of the system operation quality assessment

The quality of the system operation estimated on the basis of the features significant in the time t, $t \in \langle t_0, t_k \rangle$ could be described using so called ***Multidimensional Quality Vector***. The set of the features creates the p - dimensional assessment space. The values of the features determined for time t creates the point M'. The coordinates of the point are expressed by the vector

$[k'_{x_1(t)}, k'_{x_2(t)}, ..., k'_{x_p(t)}]$. In the multidimensional space the point is the end of the vector $\overline{WWJ}$. The vector begins in the point $[0,0,...,0]$ (Fig. 1.). It describes the quality of the system operation in time t. Similarly, the desired values of the features construct the M point. The coordinates of the point M are expressed by the vector $[k_{x_1}, k_{x_2}, ..., k_{x_p}]$. So in the assessment space the ***Model Quality Vector*** $\overline{KWJ}$ could be defined. The vector starts in the point $[0,0,...,0]$ and ends in the point M. The distance between the ends of the vectors is interpreted as a quality of the system operation $\overline{\Delta K}$.

$$\overline{\Delta K} = \overline{KWJ} - \overline{WWJ} \cdot \tag{12}$$

The wear factors change the values of the system features. So, the values of the features change in time. It could be observed as an M' point motion in period of time Δt. ***It means that the quality of system operation changes in time*** because the components of $\overline{WWJ}$ vector change on each axis in p-dimensional space in time period (t+Δt) [9].

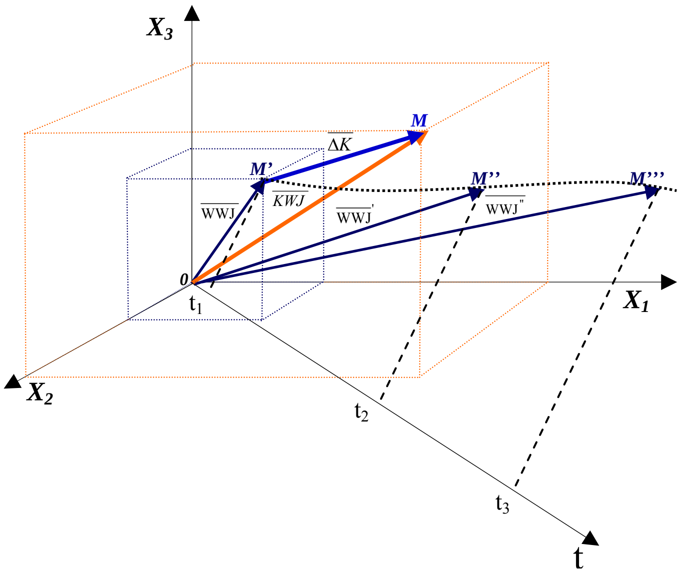

Fig. 1. Graphic interpretation of $\overline{\Delta K}$ *vector in* R^3 *space.*

The multidimensional and the model quality vectors are defined on the basis of the system features. The sharp real and the optimal values of the features determine the position of the ends of the quality vectors. In case of fuzzy interpretation of the features values the situation changes. The value of each feature is expressed by the fuzzy set defined in different domain.

$$\begin{aligned} FS_1 &= \int_{X_1} \mu_{FS_1}(X_1) \mid (X_1) \\ FS_2 &= \int_{X_2} \mu_{FS_2}(X_2) \mid (X_2) \\ &\vdots \\ FS_n &= \int_{X_n} \mu_{FSn}(X_n) \mid (X_n) \end{aligned} , \tag{13}$$

where:

FS_i - fuzzy value of the feature no. *i*,

X_i - *i* feature of the system,

μ_{FS_i} - member function of the fuzzy set no. *i*.

In such situation the ends of the multidimensional and the model quality vectors are the results of the relation of the p-dimensional extension of the flat fuzzy sets [2]. It could be expressed by the formula (14). The result of the relation constructs the area in multidimensional space. The shape of the area depends on used *T-norm* operator. The minimum and the algebraic multiplication operators are proposed as the optimal ones.

$$\begin{aligned}
&ce(FS_1; X_1 \times X_2 \times \cdots \times X_n) = \int_{X_1 \times X_2 \times \cdots \times X_n} \mu_{FS_1}(X_1) \,|\, (X_1, X_2, \cdots, X_n) \\
&ce(FS_2; X_1 \times X_2 \times \cdots \times X_n) = \int_{X_1 \times X_2 \times \cdots \times X_n} \mu_{FS_2}(X_2) \,|\, (X_1, X_2, \cdots, X_n) \\
&\vdots \\
&ce(FS_n; X_1 \times X_2 \times \cdots \times X_n) = \int_{X_1 \times X_2 \times \cdots \times X_n} \mu_{FSn}(X_n) \,|\, (X_1, X_2, \cdots, X_n)
\end{aligned}, \tag{14}$$

where:

$ce(FS_i; X_1 \times X_2 \times \cdots \times X_n)$ - n-dimensional extension of the fuzzy set no i .

The fuzzy sets implementation implicates the transformation of model and real quality vectors to *p-dimensional* areas. So, it is necessary to define the *p-dimensional* method of estimation of system operation quality.

Looking at the graphical form of the method it is possible to distinguish three cases of relative location of real system quality and model system quality areas. These areas could overlap each other, could be contiguous or separated. Obviously the biggest quality value is represented by overlapping areas and in decreasing order contiguous and separated ones. But it is necessary to develop the method of solutions ordering within each case.

In case of overlapping areas the method based on the size of the intersection volume was proposed. In case of separated areas the quality value is proportional to distance between the fuzzy sets.

5. The features weights values estimation on base of the AHP method

The values of the system features weights are calculated according to the *AHP* method (Analytic Hierarchy Process) [6].

The method compares the features between each other. It could be expressed using the matrix notation:

$$q = \begin{bmatrix} 0 & q_{1,2} & \cdots & q_{1,n} \\ 0 & 0 & \cdots & \vdots \\ 0 & 0 & 0 & q_{n-1,n} \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{15}$$

The values of the features importance creates the seven level system presented in the table:

Tab. 1. Relative preferences scale used to compare the features

Comparison of a variant to b variant	**Preference of a variant in relation to b variant**	**Number value**
a is much more significant than *b*	Strong preference of *a*	6
a is more significant than *b*	Preference of *a*	4
a is little more significant than *b*	Weak preference of *a*	2
a is as significant as *b*	No preference	0
a is little less significant than *b*	Weak preference of *b*	-2
a is less significant than *b*	Preference of *b*	-4
a much less significant than *b*	Strong preference of *b*	-6

The values of the weights are calculated according to the formula (16):

$$w_{wj} = \frac{1}{n_w} \sum_{k=1}^{n_w} q_{wjk} \,, \tag{16}$$

where:

q_{wjk} - the value of the preference the feature no. *wj,* in relation to the feature no. *k,*

n_w - amount of the features,

w_{wj} - the value of weight for *wj* feature.

Calculated values could be imprecise because of the discret and subjective characteristic of the method. It was taken into consideration by the fuzzy digits implemetation. Each value is expressed as the fuzzy triangular digit. The extent of fuzzy digit support is equal to *10%* of the rage of the weight value. The modal value of the digit is equal to the sharp value of the weight. It results the asimmetric fuzzy digits creation:

$$w_{wj}^{FS}(x) = \begin{cases} 0 & \Leftrightarrow x \le lrs \vee x \ge rrs \\ \dfrac{x - lrs}{lrk - lrs} & \Leftrightarrow lrs < x \le lrk \\ \dfrac{rrs - x}{rrs - lrk} & \Leftrightarrow lrk < x < rrs \end{cases} , \tag{17}$$

where:

$w_{wj}^{FS}(x)$ – member function of *wj* feature weight fuzzy set,

lrk – the lowest value of the fuzzy set kernel,

lrs – the lowest value of the fuzzy set support,

rrs – the biggest value of the fuzzy set support.

6. Summary

In the paper the theoretical bases of the fuzzy logic implementation in the area of the systems operation quality assessment were presented. The propositions of the fuzzy interpretation of the considered issues should be treated as a support tools in the process of the system operation quality assessment. The way of the interpretations and the usefulness of them should be verified on the basis of the exploitation experiments carried out in the real systems. Currently the proposed methods are implemented in case of complex transport system. The experiment is carried out to verify the method and to create the base for the method modification.

References

[1] Lootsma, F.A., *A model for the relative importance of the criteria in the multiplicative AHP and Smart.* European Journal of Operational Research, No 94, 19967-

[2] Łachwa, A., *Rozmyty świat zbiorów, liczb, relacji, faktów, reguł, decyzji.* Exit, Warszawa2001

[3] Pająk, M., Kalotka, J., *Biblioteka narzędziowa do tworzenia oprogramowania rozmytego i jej zastosowania.* Prace naukowe – Elektryka 1(6), ITeE, Radom 2004

[4] Pająk, M., Muślewsk,i Ł., *Rozmyty model oceny jakości działania złożonego systemu eksploatacji.* X Jubileuszowy Kongres Eksploatacji Urządzeń Technicznych, ITeE, Radom-Stare Jabłonki 2005

[5] Piegat, S., *Modelowanie i sterowanie rozmyte.* Exit, Warszawa 1999

[6] Saaty, T. L., *The Analytic Hierarchy Process, Planning, Priority Setting and Resorce Allocation.* Mc Graw-Hill, New York 1980
[7] Smalko, Z., *Podstawy eksploatacji technicznej pojazdów.* Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 1998
[8] Von Winterfeldt, D., Edwards, W., *Decision Analzsis and Behavioral Research.* Cambridge University Press, Cammbridge 1986
[9] Woropay, M., Muślewski, Ł., *Jakość w ujęciu systemowym*, Wydawnictwo Instytutu Technologii Eksploatacji, Bydgoszcz – Radom 2005